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Electric Vehicle Travelling Salesman Problem with Drone

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Outline

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Introduction

Introduction

- In United States,the transportation sector generates 28.9% of the national green house gas emissions in 2018
- The application of Electric Vehicle(EV) and Unmanned Aerial Vehicles(UAVs) is increasing dramatically as alternatives to conventional petroleum-fuel powered vehicles
- Multiple commercial companies have announced plans to use EV and UAV to accomplish delivery tasks

Problem Description

Problem Description

This research investigate a new problem called the electric vehicle travelling salesman problem with drone (EVTSP-D), where an electric truck performs deliveries with a UAV in an cooperative way.

In this problem, the EV and the UAV could perform delivery tasks simultaneously. The EV serves as the UAV hub, where the UAV can refresh its battery and be loaded with new parcels. Due to driving range limits, EV may need to visit multiple charging stations between customer visits.

Problem Description

Basic Assumptions

- All customers must be served once
- The charging station could be visited multiple times
- The EV could refresh its battery at charging stations with fixed time
- The EV and UAV share their electricity. There is a battery can be used by both vehicles. If the UAV is launched from the EV at a customer node, the electricity is deducted from the remaining battery level of the EV.

Problem Description

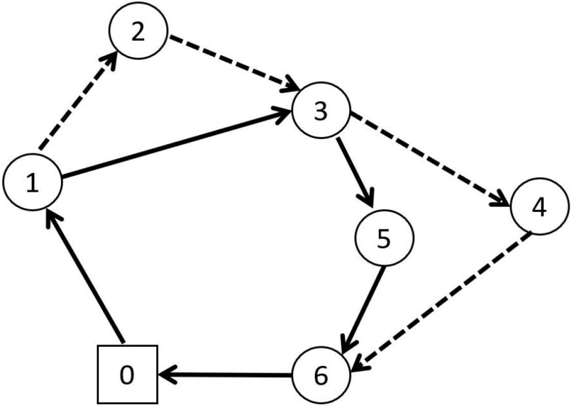

Figure: A simple representation of the EV-UAV coordinated route

Mathematical Formulation

Mathematical Formulation

Notation

Parameters

τ_{ij}	:	Travel time cost for the EV to travel from node *i* to node *j*
$\tau_{ij}^{'}$	:	Travel time cost for the UAV to travel from node *i* to node *j*
$\tau_{ijk}^{'}$	:	Travel time cost for the UAV to launch from node *i*, serves node *j* and return to node *k*
Q_d	:	Operational time limit of the UAV
Q	:	Driving time limit of the EV
S_L	:	Time needed to launch the UAV
S_R	:	Time needed to retrieve the UAV
M	:	A positive large number which is an upper bound on total travel time

Decision Variables

x_{ij}	:	Equals one if the EV travels from node i to node j and zero otherwise
y_{ijk}	:	Equals one if the UAV is launched from node i, travels to node j and returns to the EV at node k and zero otherwise
p_{ij}	:	Equals one if customer node i is visited before customer j in the EV's path and zero otherwise
u_i	:	Position of node i in the EV's path
b_i^a	:	Remaining battery charge of the EV upon arrival at node i
b_i^d	:	Remaining battery charge of the EV upon departure from node i
t_j	:	Time when the EV arrives at node j
t_j'	:	Time when the UAV arrives at node j

Objective

The objective of this problem is to minimize the final delivery time of both the EV and the UAV:

$$min\ t_{z+ms+1}$$

where $z + ms + 1$ is the total number of the nodes in the network and t_{z+ms+1} represents the time when both vehicles return to the depot

Routing Constraints:

$$\sum_{\substack{i \in N_0 \\ i \neq j}} x_{ij} + \sum_{\substack{i \in N_0 \\ i \neq j}} \sum_{\substack{k \in N_+ \\ <i,j,k> \in D}} y_{ijk} = 1 \qquad \forall j \in I \qquad (2)$$

$$\sum_{j \in N_+} x_{0j} = 1 \qquad (3)$$

$$\sum_{i \in N_0} x_{i,c+ms+1} = 1 \qquad (4)$$

$$u_i - u_j + 1 \leq (c + ms + 2)(1 - x_{ij}) \qquad \forall i \in N', j \in N_+, j \neq i \qquad (5)$$

$$\sum_{\substack{i \in N_0 \\ i \neq j}} x_{ij} = \sum_{\substack{k \in N_+ \\ k \neq j}} x_{jk} \qquad \forall j \in N' \qquad (6)$$

$$\sum_{j \in C'} \sum_{\substack{k \in N_+ \\ <i,j,k> \in D}} y_{ijk} \leq 1 \qquad \forall i \in N_0 \qquad (7)$$

$$\sum_{i \in N_0} \sum_{\substack{j \in C' \\ <i,j,k> \in D}} y_{ijk} \leq 1 \qquad \forall k \in N_+ \qquad (8)$$

$$2y_{ijk} \leq \sum_{\substack{h \in N_0 \\ h \neq i}} x_{hi} + \sum_{\substack{l \in N_0 \\ l \neq k}} x_{lk} \qquad \forall i \in N_0, j \in C', k \in N_+, <i,j,k> \in D \qquad (9)$$

$$y_{0jk} \leq \sum_{\substack{h \in N_0 \\ h \neq k}} x_{hk} \qquad \forall j \in C', k \in N_+, <0,j,k> \in D \qquad (10)$$

$$1 - (c + ms + 2)\left(1 - \sum_{\substack{j \in C' \\ <i,j,k> \in D}} y_{ijk}\right) \leq u_k - u_i \qquad \forall i \in N_0, k \in N_+, k \neq i \qquad (11)$$

Constraints (2)–(11) are associated with the routing of the two vehicles.

- constraint (2) guarantees that each customer node is visited once by either the EV or UAV
- Constraints (3) and (4) state that the EV must start from and return to the depot
- Constraint (5) and (11) are sub-tour elimination constraints
- Constraint (6) indicates that if the EV visits node *j* then it must also depart from node *j*
- Constraints (7) and (8) state that each node can launch or retrieve the UAV at most once
- Constraint (9) ensures that if there exists a UAV route <i; j; k>, then EV should travel between *i* and *k*
- Constraint (10) states that if the UAV is launched from the depot and returned to node *k*, then node k should be visited by the EV

Battery Constraints:

$$b_j^a \leq b_i^d - \tau_{ij} x_{ij} + M(1 - x_{ij}) \qquad \forall i \in N_0, j \in N_+, i \neq j \tag{12}$$

$$b_0^a = Q \tag{13}$$

$$b_i^d = Q - \sum_{\substack{j \in C' \\ j \neq i}} \sum_{\substack{k \in N_+ \\ <i,j,k> \in D}} y_{ijk} \tau'_{ijk} \qquad \forall i \in S' \cup \{0, c + ms + 1\} \tag{14}$$

$$b_i^d = b_i^a - \sum_{\substack{j \in C' \\ j \neq i}} \sum_{\substack{k \in N_+ \\ <i,j,k> \in D}} y_{ijk} \tau'_{ijk} \qquad \forall i \in I \tag{15}$$

$$b_i^a \geq 0 \qquad \forall i \in N \tag{16}$$

$$b_i^d \geq 0 \qquad \forall i \in N \tag{17}$$

Constraints (12)–(17) are associated with the battery electricity level

- constraint (12) states that if the EV travels from node *i* to node *j*, then the electricity level before arriving at node *j* is τ_{ij} less than the electricity level after leaving node *i*
- Constraint (13) ensures that when EV departs from the depot it is fully charged
- Constraint (14) states that if the EV departs from a charging station node *i* and there is a UAV route that starts at node *i*, then when EV departs from node *i*, the UAV route electricity consumption should be deducted from full-charged battery
- Constraint (15) states the same situation as constraint (14) except when node *i* is a customer
- Constraint (16) and (17) ensures that the remaining battery charge should be non-negative

Coordination Constraints:

$$t_i' \geq t_i - M\left(1 - \sum_{\substack{j \in C' \\ j \neq i}} \sum_{\substack{k \in N_+ \\ <i,j,k> \in D}} y_{ijk}\right) \qquad \forall i \in N_0 \qquad (18)$$

$$t_i' \leq t_i + M\left(1 - \sum_{\substack{j \in C' \\ j \neq i}} \sum_{\substack{k \in N_+ \\ <i,j,k> \in D}} y_{ijk}\right) \qquad \forall i \in N_0 \qquad (19)$$

$$t_k' \geq t_k + M\left(1 - \sum_{\substack{i \in N_0 \\ i \neq k}} \sum_{\substack{j \in C' \\ <i,j,k> \in D}} y_{ijk}\right) \qquad \forall k \in N_+ \qquad (20)$$

$$t_k' \leq t_k - M\left(1 - \sum_{\substack{i \in N_0 \\ i \neq k}} \sum_{\substack{j \in C' \\ <i,j,k> \in D}} y_{ijk}\right) \qquad \forall k \in N_+ \qquad (21)$$

$$t_k \geq t_h + \tau_{hk} + S_L \sum_{\substack{l \in C' \\ l \neq k}} \sum_{\substack{m \in N_+ \\ <k,l,m> \in D}} y_{klm} + S_R \sum_{\substack{i \in N_0 \\ i \neq k}} \sum_{\substack{j \in C' \\ <i,j,k> \in D}} y_{ijk} - M(1 - x_{hk}) \qquad \forall h \in N_0, k \neq h \qquad (22)$$

$$t_j' \geq t_i' + \tau_{ij}' - M\left(1 - \sum_{\substack{k \in N_+ \\ <i,j,k> \in D}} y_{ijk}\right) \qquad \forall j \in C', i \in N_0, i \neq j \qquad (23)$$

$$t_k' \geq t_j' + \tau_{ij}' + S_R - M\left(1 - \sum_{\substack{i \in N_0 \\ <i,j,k> \in D}} y_{ijk}\right) \qquad \forall j \in C', k \in N_+, k \neq j \qquad (24)$$

Constraints (18)–(24) are associated with travel time of the two vehicles

- constraints (18)–(21) ensure that the travel time and UAV range limit are correctly handled.
- Constraint (22) indicates that if the EV travels from node *h* to node *k*, its arrival time at node *k* must incorporate the its arrival time at node *h*, travel time from node h to node *k*, the UAV's launch time at node *h* and retrieve time at node *k*
- Constraints (23) and (24) are associated with the UAV's arrival time

Ordering Constraints:

$$t_k' - t_j' + \tau_{ij}' \le Q_d + M(1 - y_{ijk}) \qquad \forall k \in N_+, j \in C', i \in N_0, <i,j,k> \in D \tag{25}$$

$$u_i - u_j \le -1 + (c + ms + 2)(1 - p_{ij}) \qquad \forall i,j \in N', i \neq j \tag{26}$$

$$p_{ij} + p_{ji} = 1 \qquad \forall i,j \in N', i \neq j \tag{27}$$

$$t_l' \ge t_k' - M\left(3 - \sum_{\substack{j \in C' \\ <i,j,k> \in D \\ j \neq l}} y_{ijk} - \sum_{\substack{m \in C' \\ m \neq i \\ m \neq k \\ m \neq l}} \sum_{\substack{n \in N_+ \\ <l,m,n> \in D \\ n \neq i \\ n \neq k}} y_{lmn} - p_{il}\right) \qquad \forall i,l \in N_0, k \in N_+, i \neq k \neq l, <i,j,k>, <l,m,n> \in D \tag{28}$$

Constraints (25)–(28) are associated with ordering the two vehicles.

- constraints Constraint (25) ensures that the UAV route should be within the UAV’s flight range.
- Constraint (26) is a sub-tour elimination constraint and constrain (27) ensures the correct ordering of two different nodes
- Constraint (28) indicates that if there exists two UAV route deliveries <i; j; k> and <l;m; n> and node *i* is visited before node *l* by the EV, then node *l* must be visited after node k.

Domain Constraints:

$$t_0 = 0 \tag{29}$$

$$p_{0j} = 1 \qquad \forall j \in N_+ \tag{30}$$

$$x_{ij} \in \{0,1\} \qquad \forall i \in N_0 \tag{31}$$

$$y_{ijk} \in \{0,1\} \qquad \forall i \in N_0 \tag{32}$$

$$1 \leq u_i \leq c + ms + 2 \qquad \forall i \in N_+ \tag{33}$$

$$t_i \geq 0 \qquad \forall i \in N \tag{34}$$

$$t_i^{'} \geq 0 \qquad \forall i \in N \tag{35}$$

$$p_{ij} \in \{0,1\} \qquad \forall i \in N_0, j \in N_+, j \neq i \tag{36}$$

Solution Methods

An iterative three-step decomposition heuristic algorithm, inspired from Yurek and Ozmutlu(2018) is presented to solve the EVTSPD.

For a specific problem, given a set of customers served by the EV, the EVTSPD could be decomposed into three sub-problems

- **step 1** Solve EVTSP, with the given set of customers
- **step 2** Insert the remaining UAV nodes into the EVTSP route
- **step 3** Check if the integrated tour satisfies the battery constraints

Note that this heuristic cannot applied to large instances because the number of possible set of EV customers grows exponentially. For large instances, the number of EV customers should need to be decide before-hand.

Numerical Experiment

Computational comparison with other approaches

The first experiment compares the performance of the proposed heuristic algorithm on small and randomly generated instances with solving the problem via CPLEX, a commercial solver.

Computational comparison with other approaches

Case Name	Computational Time (s)		$cost^{opt}(s)$	$cost^{IFSA}(s)$	Optimality gap
	CPLEX	IFSA			
5C2S2R01	240	< 1	4390	4950	0.127
5C2S2R02	486	< 1	4500	4890	0.087
5C2S2R03	2103	< 1	4600	5000	0.087
5C2S2R04	2329	< 1	2720	2730	0.004
5C2S2R05	5912	< 1	4140	4960	0.198
5C2S2R06	259	< 1	6460	6650	0.029
5C2S2R07	2775	< 1	2930	3760	0.284
5C2S2R08	771	< 1	4780	6350	0.328
5C2S2R09	3038	< 1	3970	4430	0.116
5C2S2R10	6451	< 1	2780	3560	0.280
Average	**2436**	**< 1**	**4127**	**4728**	**0.153**

Computational comparison with other approaches

Case Name	Computational Time (s)				
	CPLEX	IFSA	$cost^{opt}(s)$	$cost^{IFSA}(s)$	Optimality gap
10C4S2R01	>7200	42	N/A	4210	N/A
10C4S2R02	>7200	36	N/A	5670	N/A
10C4S2R03	>7200	17	N/A	6400	N/A
10C4S2R04	>7200	32	N/A	6130	N/A
10C4S2R05	>7200	17	N/A	5630	N/A
10C4S2R06	>7200	19	N/A	5740	N/A
10C4S2R07	>7200	16	N/A	6680	N/A
10C4S2R08	>7200	10	N/A	5160	N/A
10C4S2R09	>7200	52	N/A	5260	N/A
10C4S2R10	>7200	24	N/A	6030	N/A
Average	**>7200**	**26.5**	**N/A**	**5691**	**N/A**

Real-world case study

A real-world case study, the downtown Austin network, is conducted to illustrate the effectiveness of the proposed algorithm in solving EVTSP-D with practical size.

The network contains one depot, 25 customer nodes, and 10 charging station nodes. The travel time is estimated using the Google map Python API, which is a travel time estimation on the peak hour of a typical Monday.

Real-world case study

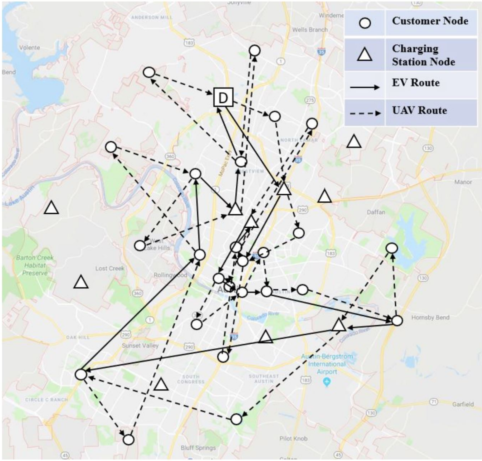

Figure: A heuristic route for the real-world case study

Conclusions & Summary

Conclusions & Summary

The main contribution of the research are

- The EVTSPD is formulated
- An efficient iterative heuristic algorithm of EVTSPD is proposed, which performs well compared to exact solution methods
- The proposed heuristic can solve problem with practical size with a reasonable computational time.

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